3. Mechanical and structural behavior

Work relative to viscoelastic solids [74N4], composite solids [74M3, 76B4], and materials undergoing phase transformation [77D6] has been reviewed in the references indicated, which we believe represent the current state of work in these fields. Past work on shock compaction of porous solids is the subject of current reviews [71H1, 74M3], but attention must also be called to certain fundamentally new work on this subject [78N3, 79N1].

3.1. Elastic solids

The continuum theory of deformation of elastic solids is old and well developed [65T1, 74T1]. In its linear version it is widely applied, but routine application of the nonlinear theory is of much more recent origin. Most of this application has been to the behavior of highly deformable materials such as rubber or to the explanation of subtle effects observed by precise ultrasonic measurements at small strain. Shock-compression experiments present an intermediate case in that materials such as vitreous silica and crystalline quartz or sapphire remain elastic to compressive strains as large as 5 to 10 per cent, exhibiting distinctly nonlinear responses over this range. In fact, nonlinear elastic effects are readily apparent in the response of a wide variety of materials to shock compression.

The stress relation obtained from an expansion of the internal energy function to fourth order in the finite strain $\boldsymbol{\eta}$ takes the form

$$t_{ij} = \frac{\rho}{\rho_R} F_{ik} F_{jl} (C_{kl} + C_{klmn}\eta_{mn} + \tfrac{1}{2} C_{klmnpq}\eta_{mn}\eta_{pq} + \tfrac{1}{6} C_{klmnpqrs}\eta_{mn}\eta_{pq}\eta_{rs} + \ldots). \tag{3.1}$$

The coefficients in this equation are functions of entropy, and are subject to a variety of thermodynamic constraints and to conditions imposed by the point symmetry of materials of interest. Various means, mostly involving very precise ultrasonic measurements at small strain, have been developed for measuring the coefficients of this equation. Some of these coefficients have been measured in shock-compression experiments and it is to this work that we now direct our attention. Relative to the ultrasonic work, shock experiments usually involve a less precise measurement at a much larger strain. Since the strains encountered in shock-compression experiments cover the entire range of elastic response, no extrapolation is involved in applications and the elastic range itself is determined.

Uniaxial strain. Plane waves of uniaxial strain can propagate in any direction into an undeformed isotropic body and in certain specific directions in anisotropic bodies [65B4]. If the 1-axes are chosen to lie in one of these allowable directions, the associated longitudinal stress is obtained from eq. (3.1) in the form

$$t_1 = \frac{\rho_R}{\rho} (C_1 + C_{11}\eta_1 + \tfrac{1}{2} C_{111}\eta_1^2 + \tfrac{1}{6} C_{1111}\eta_1^3 + \ldots), \tag{3.2}$$

where we have adopted the Voigt condensation of subscripts for symmetric tensors: $11 \to 1$, $22 \to 2$, $33 \to 3$, 23 and $32 \to 4$, 31 and $13 \to 5$, and 12 and $21 \to 6$. In dealing with the stress components and coefficients, one simply replaces the subscripts in adjacent pairs according to the above prescription, but the strains are treated according to the prescription $\eta_1 = \eta_{11}$, $\eta_2 = \eta_{22}$,

$\eta_3 = \eta_{33}$, $\eta_4 = 2\eta_{23}$, $\eta_5 = 2\eta_{13}$, $\eta_6 = 2\eta_{12}$ (similar relations hold for the linearized strain tensor S_{ij}). If, as we assume here, the material is unstressed in the reference state, the coefficient C_1 vanishes at the reference entropy. The change in entropy occasioned by passage of a shock is of the order η_1^3 (see eq. (2.19)), which means that the entropy dependence of the coefficients is negligible to the order of the expansion given except that the coefficient C_1 contributes to the highest-order term. This contribution is presumed to be small relative to errors in the determination of this coefficient in all work to date and is neglected in subsequent discussion. Using eq. $(2.4)_1$, the longitudinal stress component t_1 can be expressed in the form

$$t_1 = C_{11}S_1 + \tfrac{1}{2}(C_{111} + 3C_{11})S_1^2 + \tfrac{1}{6}(C_{1111} + 6C_{111} + 3C_{11})S_1^3 + \ldots . \tag{3.3}$$

It is noteworthy that, as pointed out by Thurston [69T1], the nonlinear material responds to finite uniaxial strain (to fourth order) as though it were linear if $C_{111} = -3C_{11}$ and $C_{1111} = 15C_{11}$. As discussed in section 2.2, a compression wave will propagate as a shock only if $C_{111} + 3C_{11} \leq 0$ (assuming the fourth-order term is negligible). This is the case with most materials and, when it prevails, the elastic coefficients can be obtained by fitting eq. (3.3) to stress-strain states obtained from shock-compression experiments conducted over the range of elastic response or from a single experiment in which the continuum of states realized in a centered decompression waveform is recorded. Graham [72G2], using data obtained by Barker and Hollenbach [70B2], has characterized Z-cut sapphire to fourth order by this method.

If $C_{111} + 3C_{11} > 0$, a centered simple wave will be produced by impact loading, and a record of this waveform suffices to determine the entire uniaxial stress-strain relation over the range of strains encountered. Vitreous silica is a material responding in this manner and its coefficients have been determined by Barker and Hollenbach [70B2] (see also [72G2]) on the basis of a simple wave analysis.

The simple wave produced by impacting vitreous silica has approximately the form of a linear ramp of velocity. When this ramp wave is used to load another elastic solid placed in contact with the vitreous silica, a measurement of the resulting smooth waveform introduced into the second material can be interpreted to yield its stress-strain response [79G2]. Because of the peculiar high-pressure properties of vitreous silica, it is capable of producing its characteristic low-pressure ramp wave even when loaded by contact with detonating high explosive (see Wackerle [62W1]). This fortuitous circumstance makes certain low-pressure measurements possible in laboratories otherwise equipped only for conducting the high-pressure hydrodynamic investigations discussed in the next section.

Shock-compression experiments carried out at stresses beyond the elastic range frequently produce a single stress-strain datum at the limit of this range (called the Hugoniot elastic limit). Knowledge of this limiting point is insufficient to fully determine the higher-order elastic properties of the material, and the measured value cannot be certified to be devoid of effects of a small amount of inelastic flow. Nevertheless, it is of interest to examine such data for evidence of contributions due to fourth-order elastic effects in cases where the third-order constants have been determined ultrasonically. To do this, we evaluate C_{111} from eq. (3.3). Neglecting the highest-order term, we find that $C_{111} = 2[(t_1 - C_{11}S_1)/S_1^2] - 3C_{11}$. Substituting tabulated values of C_{11} and the measured stress and strain at the Hugoniot elastic limit into this equation, we arrive at a result which we call C_{111}^{HEL}. A comparison of this result to other available data is shown in table 3.1. It is notable that in almost every case the absolute value of the third-order constant inferred from the Hugoniot elastic limit measurement is greater than the absolute value of the corresponding